Assessing the Uncertainty of an Estimate

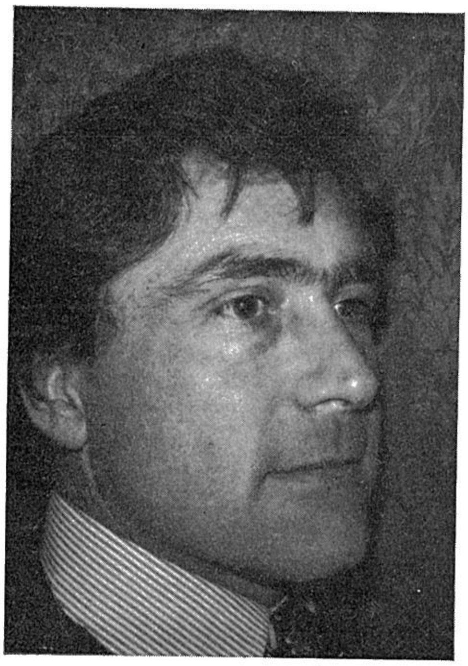
By C N Stacey, BA, ARICS

Uncertainty in cost is not easily expressed as one number. A housewife, in her household accounts, may estimate butter will cost between 40p and 42p depending on where it is bought, an uncertainty of 2p but it is possible that prices have risen since the previous week and it now costs as much as 45p. It is even possible that there has been a sudden rumour about scarcity of butter, people have been stockpiling and it now costs 80p.

There are several numbers we could give as the uncertainty in this estimate depending, for instance, on whether we are measuring uncertainty in the common occurrence or in the extremely unlikely. Butter at 40-42p is the uncertainty normally faced when shopping. The possibility of increases in cost gives a bigger uncertainty between 40 and 45p, but is less likely to happen, while scarcity and stockpiling is extremely unlikely but, if considered, it increases the uncertainty greatly, from 40p to 80p.

Which uncertainty should we choose? The choice is to some extent arbitrary, but I hope to show shortly some choices are more useful than others. It is clear that the size of the uncertainty is related to the likelihood of an event. If the less likely events are considered, the size of the uncertainty is greater.

Staticians make great use of the so-so-called "normal" distribution of probabilities. Assuming that our data matches the "normal" distribution, great strides can be made in analysing and using the data. Usually people will assume the "normal" distribution applies in default of exact information because of the simplifications it produces and, indeed, this will usually give quite accurate results even if such an assumption is wrong. We can with safety assume a "normal" distribution in our case. What is implied is that the larger the range of uncertainty for an event (eg. for buying butter) the less likely it becomes



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This paper enlarges on the details contained in my previous paper "Estimates of Uncertainty" (*Building*, 19th October 1979), and in particular examines some of the background of that paper. It concerns the calculation of a single number representing the total uncertainty in an estimate, and shows how that can be done without altering the way in which estimates are at present calculated, and with very little extra work being required.

(Figures 1 and 2 illustrate this) and also that the distribution is symmetrical about the average value (in our case not an onerous assumption).

If we plotted the graph of the likelihood of the cost being a certain amount we would certainly do it as a series of lumps or steps (as in Figure 3). It is a convenience to consider such steps being approximated by a continuous curve (Figure 4).

It should now be clear why it is difficult to specify an uncertainty using one number. If we have data as shown in Figure 4, or something similar, too narrow a range of cost in our uncertainty number would leave a fair chance of the cost being outside the range while too wide a range would be unnecessarily pessimistic.

Fortunately, established statistical methods come to the rescue. With a "normal" distribution, (or something like it) the standard measure of uncertainty is the "standard deviation".

When this measure is expressed as plus or minus £x about the central value, about $\frac{2}{3}$ of cases fall within its range (see Figure 5). If we wish to be more accurate, two standard deviations each side of the central value include 95% of all cases, and three standard deviations include 99.75% of all cases.

Measuring uncertainty in standard deviations is specific about how precise is the range of uncertainty. It also gives a helpful suggestion about how we may best present and analyse our data in estimating. Instead of expressing a range of values, eg. butter costing from 40p to 42p, the calculations are far simpler if we work on the central value (eg. butter at 41p) for calculating total costs, and then perform a separate calculation for deviation from that cost (eg. plus or minus 1p). This two part approach has the great merit of leaving unchanged present methods of calculating total costs. As I shall show later it also allows great simplification of work in calculating deviation from total cost. For the uncertainties in individual items, it should now be clear that the important part of the range of uncertainty is that where most cases will occur; it is not fruitful to try to include every conceivable extreme, for example, if you have two suppliers of bricks, one charging £54, the other charging £56 per thousand, and you may have to use the more expensive, there are the figures you should work on. The vague chance that you may need to go to someone far more expensive, or that you may acquire a job lot cheaply are irrelevant to a sensible measure of uncertainty because of their improbability.

The objection may here be raised that such an assessment of uncertainty of individual items is crude, it does not fit into the formal statistical pattern of "standard deviation" and "normal distributions". In practice such uncertainties in individual items need not be formally precise. At the

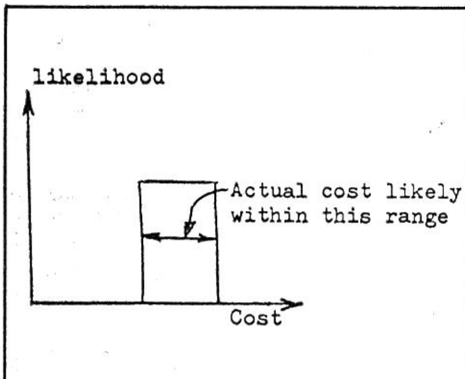

Fig 1 - Small range of uncertainty

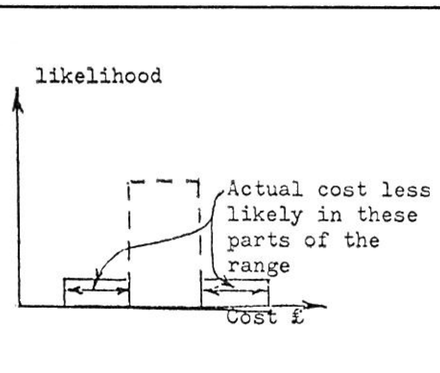

Fig 2 - Wider range takes account of less likely happenings level of individual items the notion of a standard deviation is more of value in that it shows us the spirit in which we should allow for uncertainties, ie. aim to cover the more likely part of the range of uncertainty. Of course, with uncertainties of individual items we are likely to be correct more than $\frac{2}{3}$ of the time in our assessments (as in the brick example above), so that we are being more accurate than plus or minus one standard deviation. When we later combine the uncertainties, the analysis assumes only an accuracy of plus or minus one standard deviation in individual items.

The major problem of assessing the total uncertainty of an estimate lies not in the uncertainties in individual items but in finding a reliable and simple way of assessing the effect of combining the uncertainties to give one total.

Would it now be sensible to add together all the uncertainties associated with individual items, eg. bricks ±£1,000, plasterers ±£600, excavation work ±£700, giving a total uncertainty of ±£2,700? To do so would be unduly pessimistic. The uncertainties are as likely to be minus as to be plus. Indeed the chances that they will *all* or even mostly pull one way rather than the other become less the more items we are dealing with. (We are in fact coming into another situation where the "normal distribution" of probabilities arises. With a large number of uncertainties being combined, the chances of them being mostly plus or mostly minus can be analysed using the standard deviation as a measure).

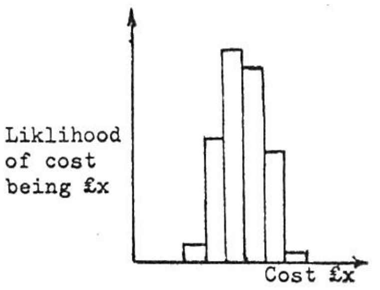

Fig 3

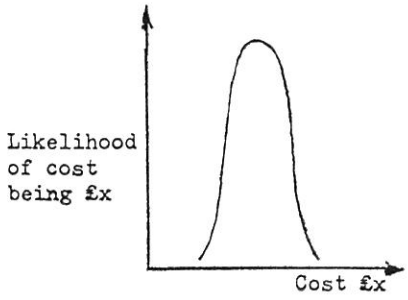

Fig 4 - Information in Fig. 3 approximated by a continuous curve

We require therefore some way of combining the uncertainties that allows a degree of overlapping (Figures 6 and 7).

We cannot allow the uncertainties to overlap entirely. Two uncertainties of ±£1,000 combined do not give a total uncertainty of ±£1,000. The total is somewhat more.

The way uncertainties should be combined to achieve a reliable measure is to calculate the square of each, add, and calculate the square root of the total. Using the example above of two ±£1,000 uncertainties, together the uncertainty is $\pm\sqrt{£(1{,}000)^2 + (1{,}000)^2} = \pm£1{,}414$. This method of combining uncertainties can be intuitively understood by considering the analogy of the uncertainty in an item with a standard deviation (which would be the measure of uncertainty if the probabilities of various values had formed a "normal distribution"). The standard deviation is calculated by squaring the values of component probabilities, summing, and calculating the square root. The method outlined above reflects this. It is as if we were calculating one enormous standard deviation from all the component probabilities of all items.

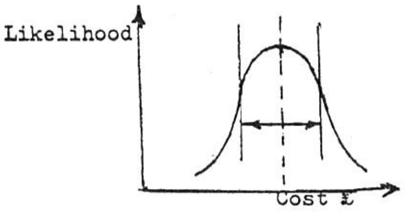

Fig 5 One "standard deviation" each side of the central value includes about $\frac{2}{3}$ of all cases.

The final total uncertainty is expressed in terms of standard deviations, ie. we may expect to come within plus or minus the total figure $\frac{2}{3}$ of the time, and within ± twice the total figure 95% of the time, and within ± three times the total figure 99.75% of the time. (This may be slightly modified if the number of items combined is small. We should then expect an accuracy more nearly comparable to the reliability of the uncertainties in the individual items. As discussed earlier this would normally be better, more accurate, than the standard deviation figures).

The overriding merit of being able to combine uncertainties as stated above is that in practice only a few important uncertainties need to be considered! These are the "dominant" uncertainties.

Suppose we combine two uncertainties, one much larger than the other, ±£100 and ±£10. Total uncertainty is $\pm£\sqrt{(100)^2 + (10)^2} = \pm£\sqrt{10{,}000 + 100} = \pm£\sqrt{10{,}100} = \pm£10\frac{1}{2}$. The smaller uncertainty has had virtually no effect on the larger, it has been swallowed up. Because of the way the total is calculated by taking squares, differences in magnitude are magnified to the extent that we may very well ignore any uncertainty which is less than a quarter the size of the largest (Figure 8).

This situation is common in estimating. A small number of uncertainties or indeed only one uncertainty, dominate, and in such circumstances we may safely ignore the myriad smaller uncertainties in the other items, their effect will be too small to influence noticeably the total uncertainty.

How should the above ideas be applied to the calculation of uncertainty in an estimate?

First we must be clear about what is the uncertainty in an item. We must consider the total effect of an item or group of related items, and not for example the uncertainty per cubic metre of an item. If there is an uncertainty in the cost of ready-mixed concrete, we must use the total uncertainty this creates in our calculations, for example if the uncertainty per cubic metre is £2, we would need to multiply the £2 per m^3 by the total quantity of concrete (from all items of such concrete) to find the total uncertainty it generated.

If there is an uncertainty about the time a project will take to complete, we must consider as a group the "Preliminaries" items which are priced on the basis of time. An increase in time affects them all en bloc.

Secondly, when presented with uncertainties in an item, we must take the central or average value for the purposes of calculating the estimate (this gives the most likely outcome for the estimate), and express the uncertainty as plus or minus about this value. In the case of the bricks cited earlier at £54 or £56 per thousand, the average

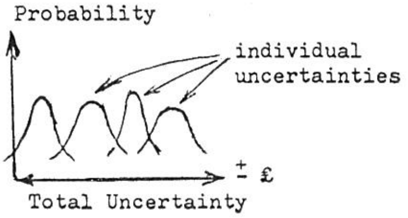

Fig 6 Adding uncertainties end on - unduly pessimistic

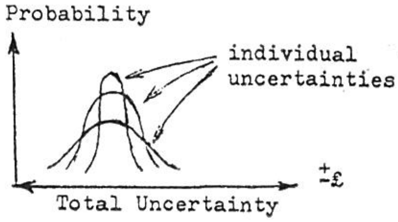

Fig 7 Combining uncertainties overlapping

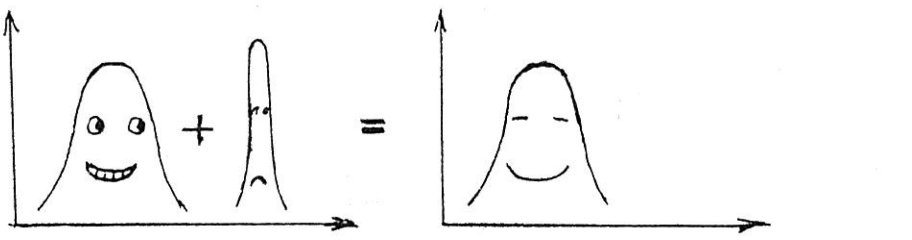

Fig 8 Dominant uncertainties swallow anything less than $\frac{1}{4}$ their size

value of £55 should be used in calculating the estimate (assuming they are both equally likely to be used), and the uncertainty of ±£1 per thousand multiplied by the quantity of bricks to give the total uncertainty in the item.

In practice it will be obvious where the major, dominant, uncertainties occur. Typically they will be among the "Preliminaries" items, such items as time for completion and major plant items, together with major items in the measured work.

When pricing the work, note where any large uncertainties lie, and on completion of the estimate calculate the uncertainties more precisely starting with the largest and continuing only as far as the uncertainties are significant to the total (ie. are dominant).

The individual uncertainties can then be combined to give one value for the uncertainty. It will resemble a "standard deviation" in that it is reasonable to expect the actual cost to be the estimated value plus or minus the uncertainty in about $\frac{2}{3}$ of cases (providing of course the estimating has been correctly done; unfortunately no calculation of uncertainty can allow for mistakes!) The situation is shown in Figure 9. Such information expresses the risk inherent in the estimate, which is clearly desirable when considering additions for overheads and profit. Such information could, in principle, also be used in assessing potential demands on the firm's capital, and possible cash flows.

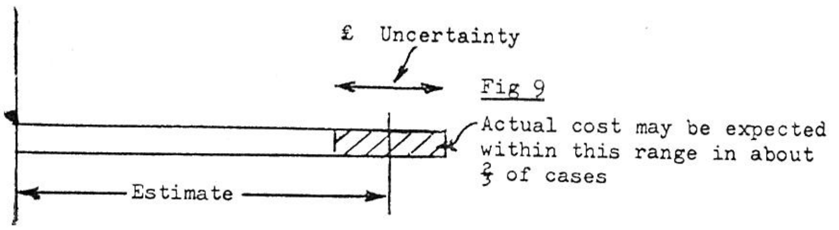

Fig 9

Report Reviews Effective Policies for Health and Safety

A review of safety policies in a variety of workplaces has been published by the Health and Safety Executive.

It was prepared by the Accident Prevention Advisory Unit (APAU) of HM Factory Inspectorate which has examined, over the past five years, the policies, organisation and arrangements for health and safety in a wide range of undertakings throughout Britain.

Companies ranging from large multinationals to small firms employing a few dozen were examined covering a broad spectrum of technology from foundries and food manufacture to electronics and building and civil engineering.

The views expressed in the review are not intended as formal advice from the HSE but the reflections and conclusions reached by the APAU, says Mr. John Locke, Director-General of the HSE in an introduction. "But", he adds, "I believe that it provides information which will be of real value to directors and managers, and particularly to those with specific responsibilities for health and safety in their organisations. I am sure it will also be of value to safety representatives, to health and safety professionals and, indeed, to all who have a concern in developing more effective arrangements for health and safety at work".

The requirement for employers of five or more persons to prepare a written policy statement is contained in the Health and Safety at Work Act 1974. Such a policy, says the review, is a formal and unique declaration of an intention to devote sufficient effort and resources to contain risk, whether to the workforce or the public.

The review emphasised that it is not possible to offer 'model' safety policies applicable to all workplaces. It suggests ways and means of introducing and managing a policy within an organisation and points out pitfalls which can prevent that policy's stated objectives from being realised at the workplace.

The practical experience of the APAU suggests that the management characteristics required to achieve high standards of health and safety are the same as those required for any other area of productvie enterprise, and the report notes that there appears to be a high correlation between those organisations which demonstrate high standards of health and safety and those which are commercially successful.

Most organisations apply considerable effort and resources to health and safety in order to stay in business, says the reviw, and in complex organisations these resources are substantial. It is important for today's managers to allocate them properly and ensure their organisation gets value for money.

"The production of a policy framework for health and safety can rationalise the problems, indicate gaps in present effort, and admit the efficient allocation of resources to the areas of greatest need", it says.

A safety policy "should not become a straight jacket to deny change or inhibit local initiative, adaptability or ingenuity by or within the organisation". It should be reviewed and updated in the light of experience, and reflect the uniqueness and special needs of the organisation for whom it is written.

"The document cannot be bought or borrowed, nor can it be written by outside consultants or inspectors. It is the decisions and actions taken by those within a workplace that ultimately determine the success or failure of attempts to influence the level of its safety performance. Valuable advice can be obtained from outside bodies, such as trade associations and HSE inspectors, but the essential tone of the policy must be established by the company which writes it", says the review.

One of the functions of the HSE, it adds, is to satisfy itself that management is competent and has the organisation and procedures to contain risk. Health and safety inspectors will increasingly require evidence of an effective safety organisation; the first piece of evidence will be the written safety policy.

An attempt is made in the review to summarise the main reasons why policies in the past, although "polished in form and style", have had less effect than was intended.

The review then attempts to identify some of the characteristics of policies which have been demonstrably successful. It was found that successful policies:

- specify that safety and health are management responsibilities ranking equally with others;
- indicate that it is the duty of the management to see that everything reasonably practicable should be done to prevent personal injury in the processes of production and in the design, construction and operation of all plant, machinery and equipment, and to maintain a safe and healthy workplace;
- indicate that it is the duty of all employees to act responsibly and to do everything they can to prevent injury to themselves and fellow workers;
- identify the main board or managing board director/directors with prime responsibility for health and safety;
- are dated as a means of ensuring they are periodically revised and signed by whoever speaks for the organisation at the highest level;
- state how and by whom their operation will be monitored.

The review moves from discussing the policy as a "statement of intent" to consider various facets of the organisation within a company which gives effect to it. It also includes, as an appendix, a suggested checklist guide of questions to probe the applicability, strengths and weaknesses of existing policy documents.

The review concludes with case studies highlighting particular aspects of safety policies and their implementation in a wide range of organisations.

'Effective Policies for Health and Safety', is available from HM Stationery Office, price £1.00 plus postage. ISBN 0 11 883254 9.